Code No: 7092/CORE

FACULTY OF SCIENCE

M.Sc. I Semester Examinations, December 2018 / January 2019

Subject: Maths/Applied Maths/Mathematics with Computer Science

Paper: I - Abstract Algebra

Time: 3 Hours | Max. Marks: 80

Note: Answer all questions from Part A and Part B. Each question carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4=32 Marks)
(Short Answer Type)

1. Give an example of a group G such that G has a normal subgroup H with both H and $\frac{G}{H}$ Nilpotent but G not nilpotent.
2. Prove that any group of order P^n, P a prime, is nilpotent.
3. Show that the group $\left(\frac{Z}{(8)},+\right)$ cannot be written as the direct sum of two non-trivial subgroups.
4. Prove that there are only two non-abelian group of order 8.
5. Let R be a commutative ring with unity. Suppose R has no non-trivial ideas. Prove that R is a field.
6. Prove that $\frac{Z[x]}{\langle x^2+1\rangle} \simeq Z[i], where\ Z[i] = \{a+b\sqrt{-1} : a,b \in z\}$ is the ring of Gaussian integers.
7. Show that 3 is irreducible but not prime in the ring $Z\left[\sqrt{-5}\right]$
8. Prove that Z[i] is a Euclidean Domain.

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) Let G be a nipolent group. Then prove that every subgroup of G and every homomorphic image of G is nilpotent.

OR

(b) Derive class equation of a finite group G.

10. (a) Let (A, +) be a finitely generated abelian group. Then prove that A can be decomposed as a direct sum of a finite number of cyclic groups.

OR

(b) (i) State and prove Cauchy's Theorem for finite abelian groups.
(ii) Prove First sylow theorem.

11. (a) Suppose R is a non-zero ring with unity. Let $I(\neq R)$ be an ideal in R. Prove that there is a maximal ideas M in R such that $I \subseteq M$

OR

(b) Suppose R is a commutative ring and P is an ideal in R. Then prove that P is a prime ideal in R if and only if "$a,b \in P,\ a \in R, b \in R \Rightarrow a \in p\ or\ b \in p$".

12. (a) If R is a UFD, then prove that R[x] is also a UFD over R.

OR

(b) (i) Prove Gauss lemma.
(ii) Prove division algorithm in polynomial rings.

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Code No: 7093/CORE

FACULTY OF SCIENCE

M.Sc. I Semester Examinations, December 2018 / January 2019

Subject: Maths/Applied Maths/ Mathematics with Computer Science

Paper- II : Mathematical Analysis

Time: 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part A and Part B. Each question carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4=32 Marks)
(Short Answer Type)

1. Prove that a set E is open if and only if its complement is closed.
2. If $\{I_n\}$ is a sequence of intervals in R^1, such that $I_n \supset I_{n+1}\,(n=1,2,3....)$, then $\bigcap_{m=1}^{\infty} I_n$ is not empty.
3. Prove that continuous image of a compact metric space is compact.
4. Let f be a monotonically increasing function defined on (a,b). Then prove that the set E of all discontinuities of f is at most countable.
5. Prove that $\int_a^{b} f d\alpha \le \int_a^{\bar{b}} f d\alpha$
6. Define Rectifiable curve.
7. State and prove M_n-test.
8. Show that the series $\sum_{n=0}^{\infty} \frac{1}{n^p + n^q x^2}$ is uniformly convergent for all values of x if p>1

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) Prove that every k-cell is compact.

OR

(b) Prove that every nonempty perfect set in R^k is uncountable.

10. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if only $f^1(V)$ is open in X for every open set V in Y.

OR

(b) Let E be a non-compact sub set of R, then prove that

(i) there exists a continuous function on E which is not bounded.

(ii) there exists a continuous and bounded function on E which does not attains its supremum.

11. (a) State and prove a necessary and sufficient condition for a bounded function f to be Riemann Stieltjes integrable on [a,b].

contd...2

(b) Suppose f is a bounded function defined on [a,b]. If α is monotonically increasing and differentiable on [a,b] such that α' is Riemann integrable on [a,b], then prove that $f \in R(\alpha)$ on [a,b] if and only if $f\alpha'$ is Riemann integrable on [a,b].

Also prove that $\int_a^b f d\alpha = \int_a^b f\alpha' dx$

12.(a) Suppose $\{f_n\}$ is a sequence of functions, differentiable on [a,b] and such that $\{f_n(x_0)\}$ converges for some point x_0 in [a,b]. If $\{f_n'\}$ converges uniformly on [a, b], then prove that $\{f_n\}$ converges uniformly on [a, b], to a function f, and $f'(x) = \lim_{n\to\infty} f'(x), (a \le x \le b)$.

OR

(b) State and prove Stone-Weierstrass theorem.

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Code No. 8061 / CORE

FACULTY OF SCIENCE

M. Sc. I – Semester Examination, December 2018 / January 2019

Subject : Mathematics / Applied Mathematics

Paper – III : Ordinary and Partial Differential Equations

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Show that $f(x, y) = xy^2$ satisfies a Lipschitz condition on the rectangle $1 \le x \le 2$ and $3 \le y \le 4$.

2 Find the complete integral of $p^2 - y^2 q = y^2 - x^2$.

3 Solve $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$ by using the method of variation of separables.

4 Find the eigen values and eigen functions of the Sturm – Liouville problem $X'' + \lambda X = 0$, $X'(0) = 0 = X'(L)$.

5 Classify the singularity of the differential equation $x^2y'' + (2 - x)y' + 5y = 0$

6 Express $f(x) = 4x^3 - 6x^2 + 3x - 7$ in terms of Legendre polynomials.

7 Express $J_2(x)$ in terms of $J_0(x)$ and $J_1(x)$.

8 Show that $H_{2n}(0) = (-1)^n \cdot \frac{(2n)!}{n!}$.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) (i) Find the complete integral of

$$x^2p^2 + y^2q^2 = z^2$$

(ii) Solve $y^2(x - y)p + x^2(y - x)q = z(x^2 + y^2)$.

OR

(b) Solve $(D^2 + DD' - 6D'^2)z = y \sin x$.

10 (a) Reduce the equation

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x^2 + y^2$$

into canonical form.

OR

(b) Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = C^2\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$

subject to the conditions $u(0, t) = u(\pi, t) = 0$ and

$$u(x, 0) = \begin{cases} x & 0 \le x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x \le \pi \end{cases}$$

11 (a) Using Frobenius method, find the series solution of
$x y'' + (1 - 2x) y' + (x - 1)y = 0$

OR

(b) (i) Show that
$(2n + 1) xP_n(r) = (n + 1) P_{n+1}(x) + nP_{n-1}(x)$
(ii) Show that $P_n(-x) = (-1)^n P_n(x)$

12 (a) Evaluate $J_{-\frac{1}{2}}(x), J_{\frac{1}{2}}(x), J_{\frac{3}{2}}(x)$ interms of $\sin x$, $\cos x$.

OR

(b) Show that $H_n(x) = (-1)^n . e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$.

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again wrt

$\frac{\partial^2 u}{\partial x^2} = XT'' - (6)$

sub eqn (4)&(5) in (1)

$XT' = c^2 X'' T$

$\frac{T'}{T} = c^2 \frac{X''}{X} = k$ (say)

$\frac{T'}{T} = k$

$T' = Tk$

$(T' - Tk) = 0$

$(D - k)T = 0 - (7)$

$\frac{c^2 X''}{X} = k$

$c^2 X'' = Xk$

$c^2 X'' - Xk = 0$

$(c^2 X'' - k)X = 0 - (8)$

case (i): If $k = 0$

from (7) $(D - 0)T = 0$

Auxillary eqn $f(m) = 0$

from (2)

$u = XT$

$u = X(x).T(t)$

Sub (9)&(10) in above eqn

$u = c(1)(c_2 + c_3 x)$ — (A)

from (8)

$c(c^2 D^2 - p^2)X = 0$

A.E $f(m) = 0$

solution of

∴ A B C are the consistency of Nature of solution.

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Code No. 8063 / CORE

FACULTY OF SCIENCE

M. Sc. I – Semester Examination, December 2018 / January 2019

Subject : Mathematics

Paper – IV : Elementary Number Theory

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 If gcd (a, b) = 1 then prove that gcd$(a^n, b^k)=1$ for all $n \geq 1$, $k \geq 1$.

2 If a prime P does not divide a then prove that (P, a) = 1.

3 If $n \geq 1$, prove that $\log n = \sum_{d/n} \wedge(d)$.

4 If f is multiplicative prove that
$$\sum_{d/n} \mu(d) f(d) = \prod_{p/n} (1 - f(P)).$$

5 Prove that $a \equiv b \pmod{m}$ if and only if a and b leave the same remainder when divided by m.

6 Show that congruence is an equivalence relation.

7 Find quadratic residues modulo 17.

8 Using Gauss Lemma find (3 / 17).

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) State and prove division algorithm.

OR

(b) Prove that there are infinitely many primes.
(c) Prove that every integer n > 1 is either a prime number or a product of prime numbers.

10 (a) If $n \geq 1$, prove that $\sum_{d/n} \phi(d) = n$.

(b) If $n \geq 1$, prove that $\phi(n) = \sum_{d/n} \mu(d) \frac{n}{d}$.

OR

(c) Prove that if both g and $f * g$ are multiplicative, then f is also multiplicative.
(d) For $n \geq 1$, prove that
$$\sum_{d/n} \lambda(d) = \begin{cases} 1 & \text{if } n \text{ is a square} \\ 0 & \text{otherwise} \end{cases}.$$

..2

11 (a) Prove that the set of lattice points in the plane visible from the origin contains arbitrarily large square gaps. That is, given any integer K > 0. There exists a lattice point (a, b) such that none of the lattice points.

$(a + r, b + s) \quad 0 < r \leq k, 0 < s \leq k,$

is visible from the origin.

OR

(b State and prove Lagrange's theorem for polynomial congruence modulo prime P.

12 (a) State and prove Gauss lemma.

OR

(b) Prove that Legendre's symbol is completely multiplicative function.

(c) If m is the number defined in Gauss lemma then prove that

$$m \equiv \sum_{t=1}^{\left(\frac{p-1}{2}\right)} \left[\frac{tn}{p}\right] + (n-1)\frac{p^2-1}{8} \pmod{2}.$$

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1212-18-505-021

Code No: 8065/CORE

FACULTY OF SCIENCE

M.Sc. I Semester (CBCS) Examinations, December 2018/January 2019

Subject: Mathematics

Paper - V : Discrete Mathematics

Time: 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part A and Part B. Each question carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4=32 Marks)
(Short Answer Type)

1. Represent the following propositions symbolically and show that it is tautology
   " If you have a current password, then you can logon to network"
   "You have a current password"
   Therefore,
   "You can logon to the network"
2. Design a logic gate for the Boolean expression
   $$f(x,y) = (x+y)(\overline{xy})$$
3. Solve the linear congruence $3x \equiv 4 \pmod 7$
4. Express gcd (414,622) = 2 as a linear combination of 414 and 622
5. State Pascal's identity and give it's combinatorial proof.
6. How many permutations of the letters ABCDEFGH contain the strings (i) ABC (ii) ABCD?
7. Show that the graph $K_{m,n}$ is planar if and only if $m \le 2, n \le 2$
8. Show that a tree with n vertices has exactly n-1 edges.

PART – B (4x12=48 Marks)s
(Essay Answer Type)

9. (a) Show that $\sim p \rightarrow (q \rightarrow r)$ is logically equivalent to $q \rightarrow (p \vee r)$

   OR

   (b) Use K-map to minimize the Boolean function
   $$f = w\,x\,\bar{y}\,\bar{z} + w\,\bar{x}\,y\,z + w\,\bar{x}\,y\,\bar{z} + w\,\bar{x}\,\bar{y}\,\bar{z} + \bar{w}\,x\,\bar{y}\,\bar{z} + \bar{w}\,x\,\bar{y}\,\bar{z} + \bar{w}\,\bar{x}\,\bar{y}\,\bar{z}$$

10. a) State and Prove Euclidian division algorithm.

   OR

   (b) Find all solutions to the system of the congruence's
   $x \equiv 2 \pmod 3$
   $x \equiv 1 \pmod 4$
   $x \equiv 3 \pmod 5$

11. (a) State and prove principle of inclusion and exclusion on n sets, $A_1, A_2, \ldots\ldots\ldots A_n$

   OR

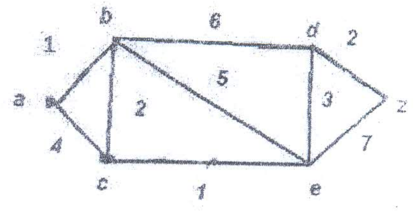
   (b) Solve the recurrence relation $a_n - 7a_{n-1} + 106a_{n-2} = 0$, $a_0 = 10, a_1 = 41$ using generating functions.

contd...2

12. (a) Show that in a connected planar graph v- e + r = 2

OR

(b) Explain Dijikra's algorithm and use it to find shortest path from a to z in the following graph:



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